



Assignment (5)

1. A first-order system is represented by the time domain differential equation $\dot{x} = x + u$.

A feedback controller is to be designed such that $u(t) = -kx$, and the desired equilibrium condition is $x(t) = 0$ as $t \rightarrow \infty$. The performance integral is defined as

$$J = \int_0^{\infty} x^2 dt,$$

, and the initial value of the state variable is $x(0) = \sqrt{2}$. Obtain the value of k in order to make J a minimum. Is this k physically realizable? Select a practical value for the gain k and evaluate the performance index with that gain. Is the system stable without the feedback due to $u(t)$?

2. An interesting mechanical system with a challenging control problem is the ball and beam, shown in Figure (1-a). It consists of a rigid beam that is free to rotate in the plane of the paper around a center pivot, with a solid ball rolling along a groove in the top of the beam. The control problem is to position the ball at a desired point on the beam using a torque applied to the beam as a control input at the pivot.

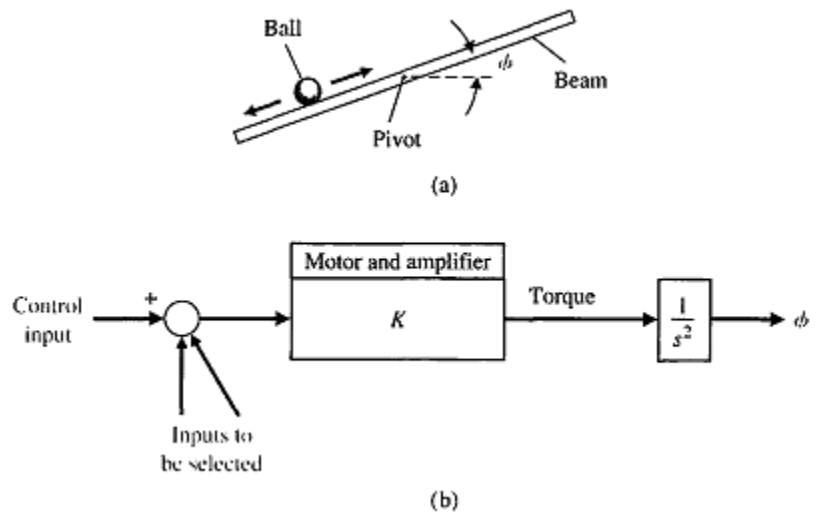


Fig. 1. (a) Ball and beam.(b) Model of the ball

A linear model of the system with a measured value of the angle ϕ and its angular velocity $d\phi/dt = \omega$ is available. Select a feedback scheme so that the response of the closed-loop system has an overshoot of 4% and a settling time (with a 2% criterion) of 1 second for a step input.



3. Consider the single-input, single-output system is described by

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t)$$
$$y(t) = \mathbf{C}\mathbf{x}(t)$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -16 & -8 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0 \\ K \end{bmatrix}, \mathbf{C} = [1 \ 0].$$

(a) Determine the value of K resulting in a zero steady-state tracking error when $u(t)$ is a unit step input for $t \geq 0$. The tracking error is defined here as $e(t) = u(t) - y(t)$.

4. Consider the device for the magnetic levitation of a steel ball, as shown in Figures (1) (a) and (b). Obtain a design that will provide a stable response where the ball will remain within 10% of its desired position. Assume that y and dy/dt are measurable

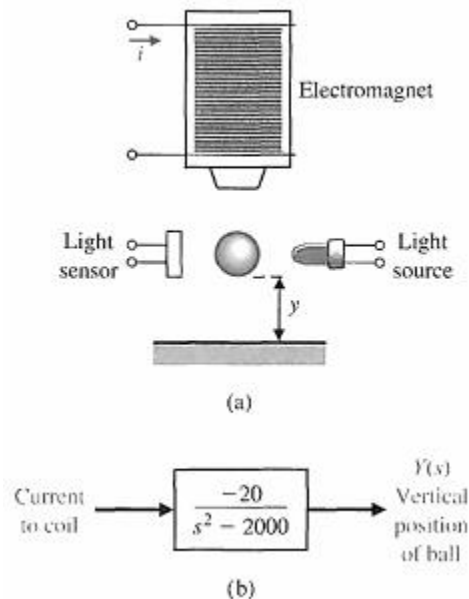


Fig. 2. (a) The levitation of a ball using an electromagnet, (b) The model of the electromagnet and the ball.



5. The goal is to design an elevator control system so that the elevator will move from floor to floor rapidly and stop accurately at the selected floor Figure (3). The elevator will contain from one to three occupants. However, the weight of the elevator should be greater than the weight of the occupants; you may assume that the elevator weighs 1000 pounds and each occupant weighs 150 pounds. Design a system to accurately control the elevator to within one centimeter. Assume that the large DC motor is field-controlled. Also, assume that the time constant of the motor and load is one second, the time constant of the power amplifier driving the motor is one half second, and the time constant of the field is negligible. We seek an overshoot less than 6% and a settling time (with a 2% criterion) less than 4 seconds

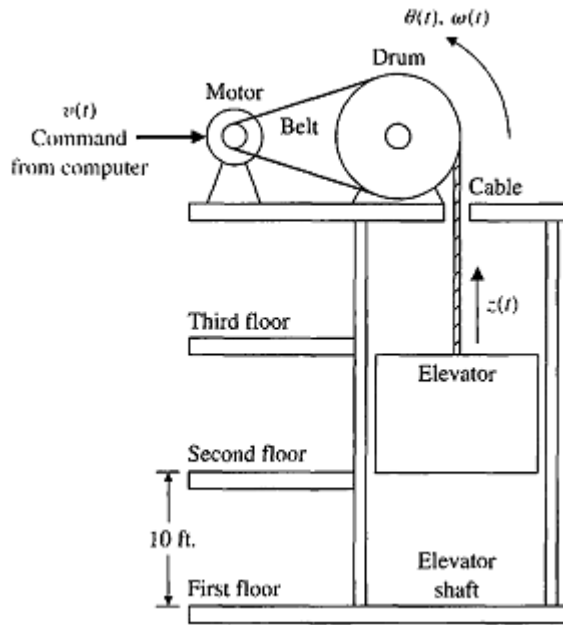


Fig. 3. Elevator position control.